Notes:

- *Time allowed is 4 hours*
- *Begin each question on a new sheet of paper*
- *All your working and methods must be shown clearly*
- *Each question is worth 10 marks*
- *The TI–89, TI–92 or Voyage 200 may be used.*
- *No functions, variables or programmes may be stored under VAR-LINK*
- *Your copy of the formula collection, the probability tables and the list of TI-Functions may be used.*

1a Given the function f(x) = $x^2 \cdot e^{\frac{-x^2}{2}}$ x >0

(i) Sketch the curve for $0 \le x \le 5$ **(1)**

(ii) Calculate the co-ordinates of the maximum point of the curve in your sketch and describe briefly how this point is found. **(2)**

(iii) Find the width of the curve, where width means the difference in the x values of the two points of inflection on the curve. **(1)**

(iv) Calculate to 3 decimal places (dp) the area between the curve and the x axis . **(1)**

(v) Investigate the relationship between your answer to part (iv) and the number √π **(1)**

1b Fig 1 shows the curve y = x (4 –x) together with a straight line. This line cuts the curve at the origin O and the point P with x co-ordinate k, where $0 \le k \le 4$

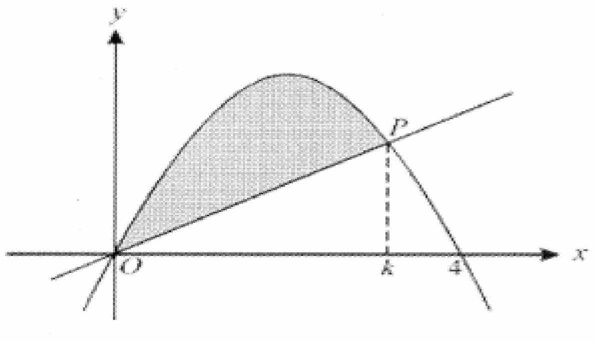


Fig 1

(i) Show that the area of the shaded region between the curve and the line is equal to $\frac{k^3}{6}$ **(2)**

(ii) Find, correct to 3 decimal places the value of k for which the area of the shaded region is half the total area under the curve between x =0 and x =4 **(2)**

2a As a substance cools, its temperature T(°C) is related to time x (minutes) for which it has been cooling by the relationship

$$T = 20 + 60\,e^{-0,1x} \quad x \geq 0$$

(i) Find the value of T when the substance first starts to cool. **(1)**
(ii) Sketch the graph of T against x and comment on its shape. **(1)**
(iii) Find the value of x (2 dp) at which T = 60°C **(1)**
(iv) Find the value of T at which the temperature is decreasing at the rate of 1,8°C per minute. **(2)**

2b A technical manual of ship design contains the following information

Speed **V** (knots)	6	7	8	9	10	11	12	15
Power **P** (bhp)	70	80	160	300	515	840	1800	6500

(i) Using the attachment to this question (page 3), plot the graph of ln P against ln V **(2)**

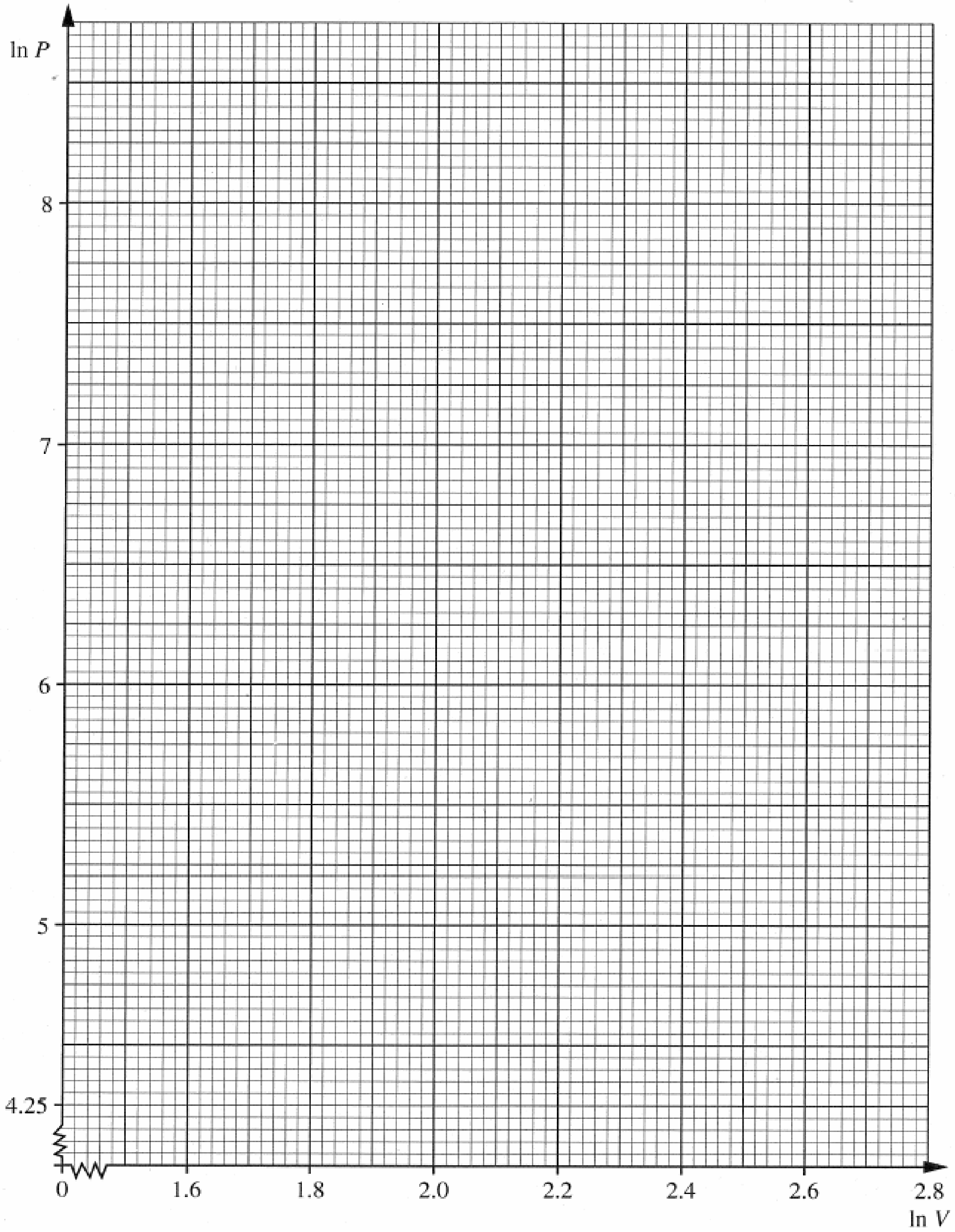
(ii) State with reasons the range of values for which the equation

$P = kV^n$ provides a good model for the relationship between P and V **(1)**

(iii) Estimate the values of k and n and explain briefly how you obtained them. **(2)**

Attachment to question 2b

V	6	7	8	9	10	11	12	15
$\ln V$	1.79		2.08		2.30		2.48	
P	70	80	160	300	515	840	1800	6000
$\ln P$	4.25		5.08		6.24		7.50	



3a
Points A and B have coordinates (3/2/4) and (4/4/-3) respectively. A model aircraft takes off from point A and flies in a straight line l_1. The equation of line l_1 is:

$$\mathbf{r} = \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ 1 \\ 1 \end{pmatrix}$$

(i) Show that l_1 is perpendicular to AB **(1,5)**

Another model aircraft takes off from point B in a line l_2 whose equation is;

$$\mathbf{r} = \begin{pmatrix} 4 \\ 4 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$$

(ii) Show that the paths of the two aircraft cross and give the coordinate of the point of intersection. **(2)**

3b

Test drilling in the Namibian desert has shown the existence of gold deposits at (400 /0 /-400), (-50/ 500/-250), (-200/ -100/ -200) where the units are in metres. The x axis points east, the y axis points north and the z axis points up. Assume that these deposits are part of the same seam contained in plane π_1.

(i) Show that the equation of plane π_1 is (x +3z + 800) **(1,5)**

(ii) Find the angle at which π_1 is tilted to the horizontal. **(2)**

The drilling positions (400/ 0/ 3), (-50/ 500/ 7), (-200/ -100/ 5) are on the desert floor. Take the desert floor as a plane π_2,

(iii) Find the equation of π_2 **(1,5)**

(iv) Find the equation of the line where the plane containing the gold seam intersects the desert floor. (If you have not found an answer for (iii), use the following equation for π_2: 14x – 15y + 3450z – 15950) **(1,5)**

4a

For any married couple who are members of a certain tennis club, the probability that the husband has a degree is $\frac{3}{5}$ and the probability that the wife has a degree is $\frac{1}{2}$. The probability that the husband has a degree given that the wife has a degree is $\frac{11}{12}$.

i) A married couple is chosen at random. Show clearly that the probability that both of them have a degree is $\frac{11}{24}$ **(1)**

ii) Draw a Venn diagram to illustrate these data. **(2)**

Find the probability that:

iii) only one of a couple has a degree **(1)**

iv) neither of them have a degree. **(0,5)**

Two married couples are chosen at random.

v) find the probability that only one of the husbands and only one of the wives have degrees. **(2)**

4b Statistical tables are to be found in the appendix page 7

A large company is considering introducing a new selection procedure for job applicants. The selection procedure is intended to result over a long period in equal numbers of men and women being offered jobs. The new procedure is tried with a random sample of applicants and 17 of them, 13 women and 4 men are offered jobs.

i) Carry out a suitable test at the 5% level of significance to determine whether it is reasonable to suppose that the selection procedure is performing as intended. You should state the null and alternative hypotheses under test and explain carefully how you arrive at your answer. **(2,5)**

ii) Suppose now that of the 17 applicants offered jobs, w are women. Find all the values of w for which the selection procedure would be judged acceptable at the 5% level. **(1)**

5

This question concerns the function

$$f(x) = \frac{1}{(x+k)^n}$$ where $k \geq 0$ and n = 1 or 2

a) Consider f(x) where k = 1 and n =1 . Between the curve and the x axis lies an isosceles triangle OBC. (OC = BC). Where vertex O lies at the origin (0/0), vertex B (3 / 0) and C touches the curve.

(i) Sketch the curve and triangle OBC clearly. **(2)**

(ii) Calculate the area of triangle OBC **(2)**

b) Now consider f(x) where $k \geq 0$ and n =2. You will investigate whether a triangle similar to OBC with O (0 /0), B (x / 0), C touching the curve, exists for every value of k and, if so, whether the area of the triangle is a maximum or minimum. Here is a suggested method:

Write an equation for the triangle's area A in terms of x and k as a parameter
Use calculus to find how A changes with x and whether the area is a maximum or minimum . Then find the relationship of x to k. **(3,5)**

c) Investigate the situation as in part (b) for f(x) with n = 1 $k \geq 0$

i) Do triangles such as OBC exist for all values of k? If so, are these triangles maximum or minimum in terms of area? **(2,5)**