


Matura Examinations 2024 – Mathematics

Classes: 4Be, 4GL, 4We

Teachers: BtT, PaS

- Note: You have four hours to complete the examination.
Begin each question on a new sheet of paper.
- Permitted materials: TI-30X Pro calculator
The *Fundamentum Mathematik und Physik*, without notes
English-German dictionary


All questions labeled with the symbol  are to be solved **by hand**. For these questions, only the basic functions of your calculator are permitted. To attain full marks in these questions, you should not use commands such as num-solv, poly-solv or the numerical calculation of derivatives or integrals.

Question 1: Vector Geometry (12.5 marks)

Consider the line l with the vector equation

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \\ 19 \end{pmatrix} + s \cdot \begin{pmatrix} 1 \\ 4 \\ -8 \end{pmatrix}$$




and the two points $A(6, 9, -5)$ and $B(9, 6, 2)$.

- (a) Verify whether the point A lies on the line l . (1 P.)
- (b) The plane P_1 is perpendicular to the line l and contains the point B . Find the Cartesian equation of the plane P_1 . (2 P.)
- (c) Find the angle between the plane P_1 and the plane $P_2 : 4x + 3y - 4 = 0$. (1.5 P.)
- (d) Provide the coordinates of the points on the line l which have a distance of $d = 3$ units from the plane P_2 . (3 P.)
- (e) The line l intersects the yz -plane at point C . Calculate the area of the triangle ABC . (2.5 P.)
-  (f) For which values of a is the following line k_a orthogonal to the line l ? (2.5 P.)
- Note: It is not required that the lines l and k_a intersect.

$$k_a : \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 12 \\ 2 \end{pmatrix} + u \cdot \begin{pmatrix} a^2 \\ 1 - a \\ 2 \end{pmatrix}$$

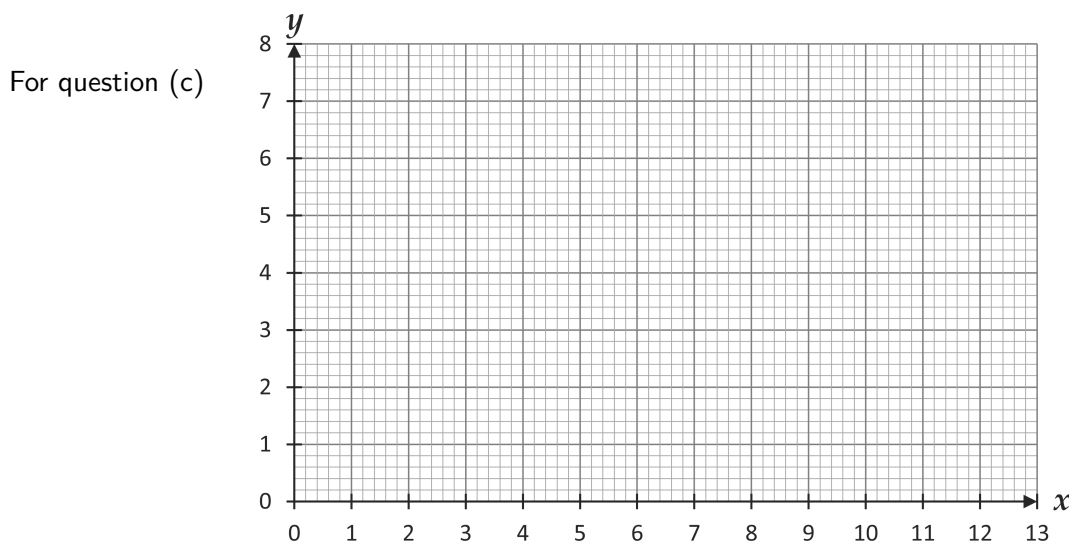
Question 2: Calculus (14 marks)

The polynomial function f is given by $f(x) = \frac{1}{36}x^3 - \frac{1}{2}x^2 + \frac{9}{4}x$.

-  (a) (a₁) Find the zeros of f . (1.5 P.)
- (a₂) Show that f has exactly one point of inflection at $\left(6, \frac{3}{2}\right)$. (2 P.)
-  (b) The function f has exactly two turning points which are $M_1(3, 3)$ and $M_2(9, 0)$.
Determine by calculation whether these points are minima or maxima. (1 P.)
- (c) Draw the graph of f in the coordinate system given below. (1 P.)
-  (d) Calculate the area which is bound by the graph of f the x -axis and the vertical line $x = 6$.
The area of interest is on the left of the line $x = 6$. (1.5 P.)
- (e) Give an equation of the tangent at the point of inflection. (2 P.)

If you could not find the equation of the tangent in the point of inflection in question (e), then use the line equation $t(x) = -\frac{3}{4}x + 6$ for question (f).

- (f) A triangle is bound by the y -axis, the tangent in the point of inflection and the line passing through the origin and the point of inflection. The graph of f divides this triangle into two parts.
Calculate the areas of both parts. (2.5 P.)
- (g) The function h is formed from f according to the equation $h(x) = \frac{f(x) + 1}{x^3}$.
- (g₁) Determine the equations of all the asymptotes of h . (1.5 P.)
- (g₂) Discuss the behaviour of the function h on both sides of the singularity. (1 P.)



Question 3: Calculus (11 marks)

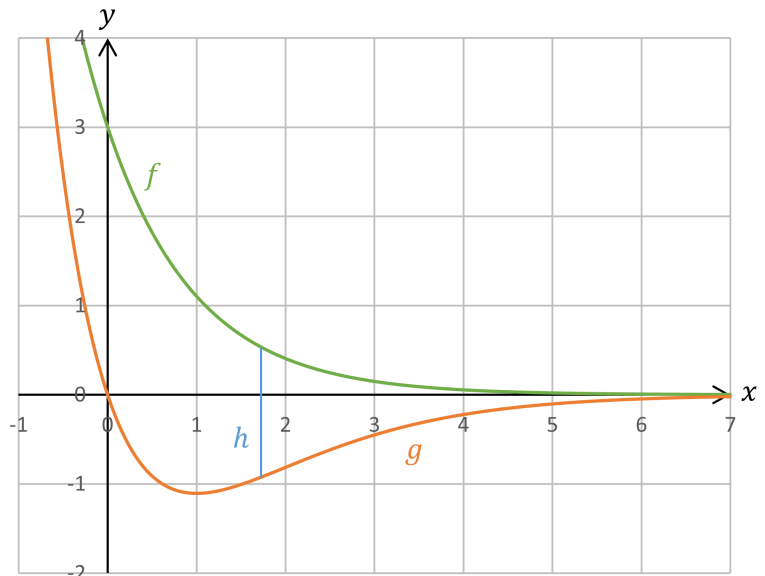
Consider the two functions



$$f(x) = 3e^{-x} \quad \text{and} \quad g(x) = -3x \cdot e^{-x},$$

where e is Euler's number.

The graphs of both functions are shown in the following figure.

The blue line segment labeled h refers to task (d).



-  (a) Calculate the point of intersection of f and g . Provide the exact coordinates. (2 P.)
- (b) Differentiate both functions f and g and simplify until you reach:
 $f'(x) = -3e^{-x}$ and $g'(x) = 3e^{-x} \cdot (x - 1)$
 Provide step-by-step solutions. (2 P.)
- (c) At what angle α do the curves intersect? Round the angle to two decimal places. (2 P.)
Note: If you could not find the point of intersection in task (a) use $x = -1.5$ for this task.
-  (d) Find the value of x with $x \geq -1$ for which the vertical distance h between the two graphs is maximal (see figure). Prove that it is a maximum. (3 P.)
- (e) The graph of f is rotated around the x -axis. The solid of revolution for the x interval $[0; b]$ has a volume of 14.
 Calculate the right boundary b , rounded to two decimal places. (2 P.)

Question 4: Probability (11.5 marks)

The question consists of three independent sub-questions.

Question 4.1

In a wardrobe, there are five trousers, 3 blue and 2 black. Additionally, there are 7 T-shirts, 2 white, 1 black, and 4 coloured ones.

One of each (trousers and T-shirt) is chosen randomly.

- (a) Draw a tree diagram to illustrate all possible outcomes. Calculate the probability for each path. (2 P.)
- (b) What is the probability that both garments are black? (0.5 P.)
- (c) How big is the probability that no white T-shirt is chosen? (0.5 P.)

Question 4.2

A random number generator provides the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 with equal probability.

- (a) Three numbers are generated. What is the probability that they are the same? (1 P.)
- (b) 15 numbers are generated. What is the probability that exactly five of those are the number 9? (1.5 P.)
- (c) Two consecutively generated numbers are added and the sum of the pair is investigated. 20 such pairs are generated.
What is the probability that at most two of those 20 pairs have the sum 4? (1.5 P.)

Question 4.3

At a school, students must choose 4 subjects which are listed in 3 different categories. Category A is Science with 3 subjects, category B is Languages with 4 subjects and category C contains 2 subjects (sport and theatre).

- (a) In how many ways can the 4 subjects be chosen if there are no restrictions? (1 P.)
- (b) How many possibilities are there if students must choose at least one subject from each category? (1.5 P.)
- (c) 84% of the participants of the theatre course are female. 70% of all females choose theatre. Only 20% of the rest of the students choose theatre. Find the percentage of female students at the school. (2 P.)

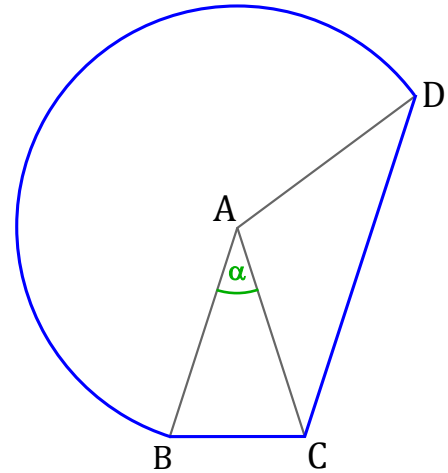
Question 5 (12 marks)

5.1 Trigonometry

A metalworker cuts out a piece of sheet metal as shown in the figure. The three points B , C and D lie on a circle with centre A and radius 5 cm.

The line segments \overline{AB} and \overline{CD} are parallel.

- (a) Calculate the perimeter of the piece of sheet metal for $\alpha = 30^\circ$. (3.5 P.)



- (b) However there is a restriction. The angle α must satisfy the equation:

$$10 \cdot \sin(2\alpha - 10^\circ) = 7$$

Find all possible angles of α for $0 < \alpha < 90^\circ$. (2.5 P.)

5.2 Solar cooker

Since the position of the sun changes throughout the day, the performance of a solar cooker varies.

Let t be the time in hours elapsed since sunrise at 6:00 AM. The power P of a solar cooker in watts is described by the following function for $t \in [0; 6]$, i.e., for the time between 6:00 AM and 12:00 PM.

$$P(t) = 600 \cdot (1 - e^{-0.6t})$$



- (a) Draw the graph of the function P in the coordinate system on the next page. (1 P.)

- (b) At what time does the power of the solar cooker exceed 450 W? Provide the time of day to the nearest minute. (2 P.)

- (c) Calculate the total energy in watt-hours Wh that the cooker has absorbed during the entire morning (6:00-12:00). Remember that power is energy per unit time. (1.5 P.)

- (d) The solar cooker is a very interesting ecological alternative despite many disadvantages (cooking only takes place outdoors, weather dependency, etc.).

After how much time does the number of solar cookers in the world double, if the number increases by 5% each year? (1.5 P.)

Attachment

