

Matura Exams 2025 - Written Mathematics

Classes: 4Ba, 4Bb, 4Be, 4GLW, 4IM, 4KSW, 4SI, 4W, 4We, 4WZ

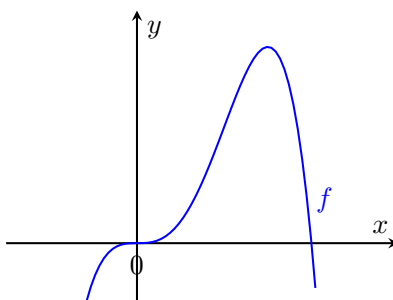
Teachers: BtT, FrC, HuR, KiA, KrD, MoM, PeM, PrG, RoL, SuF

Note:	You have four hours to complete the examination. Begin each question on a new sheet of paper.
Permitted Materials:	TI-30X Pro MathPrint Calculator The Mathematics and Physics <i>Fundamentum</i> , without notes English-German dictionary

For each question, you are expected to show a clear method. In part-questions which are to be solved **by hand**, only the basic functions of your calculator are permitted. To attain full marks in such questions, you must not use commands such as: *num-solve*, *poly-solve*, *sys-solve*, *DotProduct*, *CrossProduct* or the numerical calculation of derivatives or integrals. The part-questions which must be solved **by hand** will be indicated with a  symbol.

Question 1: Calculus (12 Points)

The polynomial function f with equation $f(x) = -\frac{1}{4}x^4 + x^3$ is given. A section of the graph of function f is shown below.



-  Calculate the full coordinates of all optimal points ("Extremalpunkte") for the graph of f . Determine clearly whether their type is "Maximum" or "Minimum". (3.5 P.)
-  The point $W(2|4)$ lies on the graph of f . Show that W is a point of inflexion. (1 P.)
- The y -axis, the tangent line at point W , together with the normal line at point W form the sides of a triangle. Find the area of this triangle. (3 P.)

An additional function $g(x) = x^3 - 2x$ is introduced.

-  The graphs of functions f and g form a bounded region, R_1 . Calculate the area A_1 of this region. (3 P.)
- A real number, p , has a value found in the interval $0 \leq p \leq 4$. The region, R_2 , is bounded by the x -axis, the lines $x = 0$ and $x = p$, and finally, by the curve of f . Given that the area of region R_2 is equal to 10 square units, find the value of p . (1.5 P.)

Question 2: Vector Geometry (13 Points)

A triangle with vertices $A(5|8|3)$, $B(11|0|3)$ and $C(x_C|y_C|z_C)$ is given. The mid-points of its three sides $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ are points $X(8|4|3)$, $Y(5|-2|3)$ and $Z(2|2|3)$ respectively (see Diagram 1).

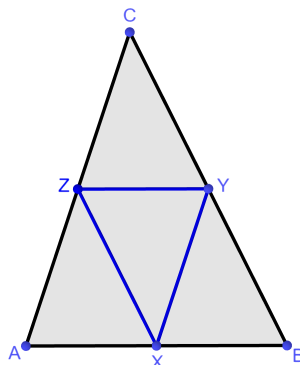


Diagram 1

- Using the information provided, show by a calculation method that vectors $\overrightarrow{AB}$ and $\overrightarrow{ZY}$ are parallel. (1 P.)
- Calculate the full coordinates of point C . (1 P.)
- Calculate the area of the triangle XYZ . (1.5 P.)

The triangle ABC is now folded at the three lines which join the midpoints X , Y and Z . As a result, the three vertices A , B and C will meet to form the summit-point, S , of a pyramid whose base is the triangle XYZ (see Diagram 2).

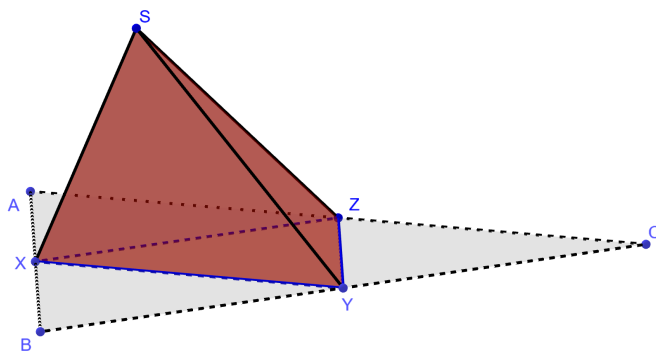


Diagram 2

- During the folding process - as point A moves up to reach point S - the point A will remain within a plane Π_A that is perpendicular to the folding edge $\overline{XZ}$. Show that a Cartesian equation for plane Π_A could be expressed as:
$$\Pi_A : 3x + y - 23 = 0$$
 (1 P.)
- By a similar folding process, point B moves up to reach point S while remaining within plane Π_B throughout the movement. Find a suitable Cartesian equation for plane Π_B . (1.5 P.)

This question continues on Page 3.

Question 2 continues here.

- (f) The line of intersection of the two planes Π_A and Π_B will be called g_s , since it contains the point S . Find a vector equation for the line g_s . (2 P.)

Advice: If you were unable to obtain a Cartesian equation for plane Π_B in part (e), then you should continue using the replacement plane $3x - 4y - 13 = 0$.

A student builds a model of this pyramid, and measures the coordinates of the point S , accurate to one decimal place. His result was:

$$S(7.0|2.0|7.5)$$

For any further calculations involving point S , you should use these rounded coordinates.

- (g) Calculate the acute angle between the pyramid's base area XYZ and its face XZS . Round your answer to one decimal place. (2.5 P.)
- (h) Calculate the volume of the pyramid $XYZS$. (1.5 P.)
- (i) What is the ratio between the volumes of pyramid $XYZS$ and pyramid $ABCS$? Your answer should be an explanation in words, *without* a calculation. (1 P.)

Question 3: Probability (10 Points)

A teacher of German gave the same homework to each of her three classes. The task was to write a one-page essay on a chosen theme. For information, the class sizes were as follows:

Class Name	4A	4W	4Z
Class Size	23	18	14

After collecting the homeworks from all of the students, the teacher took them home to mark (grade). To her surprise, all of the papers in the pile were mixed together in a random order. Could this have been by accident? Or was it a practical joke?

- (a) In how many different arrangements could the students work now be found? (0.5 P.)

In order to save time, the teacher shrugged her shoulders and began marking from the top of the unsorted pile of essays.

- (b) What is the probability that the first essay that the teacher corrected was not written by a student of Class 4W? (1 P.)
- (c) What is the probability that the first three corrected essays all came from different classes? (1.5 P.)

In the era of Artificial Intelligence (AI), chatbots can be used to produce an essay at the “press of a button”. A recent survey has shown that 11% of students no longer prepare their own homework, but instead, use AI to write it for them. It is, therefore, reasonable to assume that 11% of all homeworks handed in at Gymnasium Liestal have been generated by Artificial Intelligence.

- (d) What is the probability that, of five randomly-chosen essays, all were produced using Artificial Intelligence? (0.5 P.)
- (e) What is the probability that, of five randomly-chosen essays, exactly three were produced using Artificial Intelligence? (1.5 P.)
- (f)  What would be the smallest number of essays that need to be collected, in order to reach 99% confidence that at least one of them was produced with AI? (2.5 P.)
- (g) A company offers an AI Recognition Service (AIRS) to teachers. The company claims that it can correctly identify with 90% accuracy, an AI-produced essay as “Generated”. The company claims the same success rate with identifying students’ own work as “Not Generated”.

What is the probability that an essay which was put through this Recognition Test, and was then labelled as “Generated”, was in truth written solely by the student? (2.5 P.)

Question 4: Hypothesis Testing (6.5 Points)

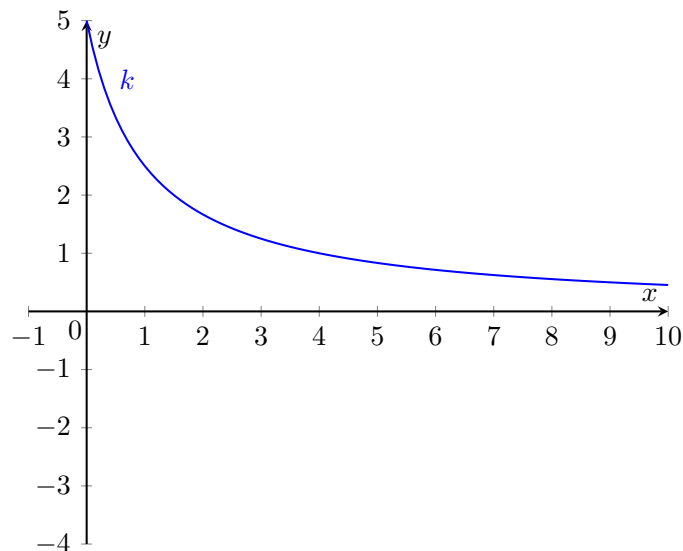
Many children die each year from Malaria. Most of these deaths occur as a result of the first infection, since they have not yet developed a natural immunity to the parasite. In several African countries, large-scale studies have been carried out to test the effectiveness of a new “malaria vaccine”. As a rule, 40% of children in these countries where no vaccine is available will develop Malaria at some time during a single year.

For one of these studies, a group of 310 children were vaccinated with the new “malaria vaccine” and then observed for a full year to see how many of them developed Malaria.

- (a) How many Malaria infections would you expect to find in this group of children if the vaccine had no effect? (0.5 P.)
- (b) The manufacturer claims that the vaccine could reduce infection levels for a significant number of children. It is agreed, that the vaccine can be considered as 'effective' if fewer than 115 children from this study group develop Malaria during the year.
 - i. What is the probability that, at the end of the year, the vaccine will falsely be considered to have been “effective” (Type 1 error)? (1.5 P.)
 - ii. What is the probability that the vaccine will falsely be seen as “ineffective” (no effect) if the manufacturer claims that the vaccine will reduce the probability of Malaria within the group to 30% (Type 2 error)? (2.5 P.)
- (c) At the end of the year, 110 of the fully vaccinated study group have developed Malaria. Using a significance level of $\alpha = 5\%$, can this vaccine still be accepted as “effective”? Justify your decision. (2 P.)

Question 5: Calculus - Optimisation (8 Points)

A section of the graph of function $k(x) = \frac{5}{x+1}$ is shown below.



Using a value $u > 1$, we are able to define a right-angled triangle PQR , formed by points:

$P(u|k(u))$ as a chosen point on curve, k , together with points $Q(1|0)$ and $R(u|0)$.

This triangle is rotated around the x -axis creating a standard cone.

(a) Using the value $u = 4$, sketch such a cone onto the diagram shown above. (0.5 P.)

(b)  Show that the volume V of this type of cone, in terms of u , can be expressed as the function: (2.5 P.)

$$V(u) = \frac{25\pi}{3} \cdot \frac{u-1}{u^2+2u+1}$$

(c)  Calculate the exact value of u that produces such a cone with the maximum volume. What is the value of this maximum volume? (5 P.)

Advice: You are not required to prove that the volume that you have found is, in truth, a maximum value.

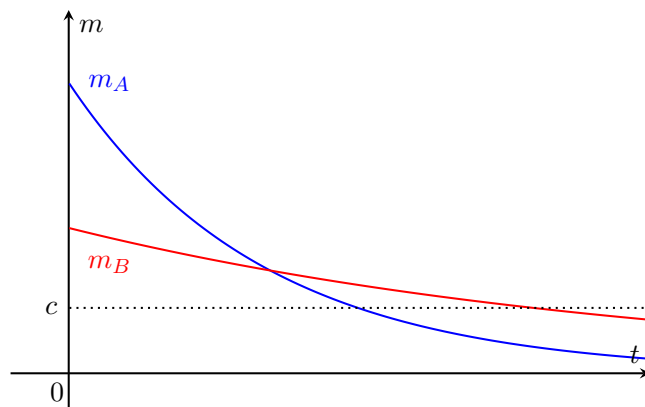
Question 6: Exponential Decay (7.5 Points)

In a laboratory, the radio-active decay of two substances, A and B , are observed. The symbol m representing the mass (in grams) of each substance, and t represents time (in hours) measured from the start of the observation period.

The radio-active decay of these two substances are shown by the following exponential functions where e is Euler's number:

$$m_A(t) = 400 \cdot e^{\frac{-3t}{10}} \quad \text{und} \quad m_B(t) = 200 \cdot e^{\frac{-t}{10}}$$

The following diagram (not drawn to scale) indicates the graphs of the two functions.

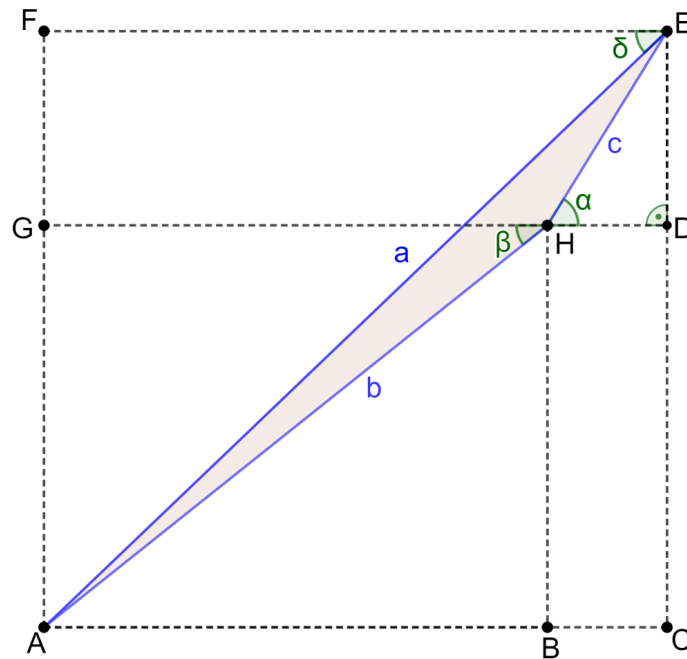


- Show that, at the start of the observation period, the mass of radio-active substance B is exactly half of the mass of radio-active substance A . (0.5 P.)
- How long will it take for the radio-active mass of substance A to fall to half of its original radio-active mass? (1.5 P.)
- After how many hours of observation will the radio-active mass of both substances be equal? (2 P.)
- For what value of time, t , will the decay rate for substance A be exactly $40 \frac{\text{grams}}{\text{hour}}$? (1.5 P.)
- The observation period ends when a "critical mass" of c grams is reached. This "critical mass", is the same value for both substances and occurs for radio-active substance B exactly one hour later than for radio-active substance A . Calculate the value of the "critical mass" c . (2 P.)

Question 7: Trigonometry (5 Points)

Rectangles $ABHG$ and $ACEF$ are shown in the following diagram. Using their diagonals, $\overline{AH}$ and $\overline{AE}$, the triangle AHE can be formed. The following information is given:

$$\alpha = 60^\circ, \beta = 38^\circ, \overline{AH} = 667.5 \text{ mm}, \overline{HE} = 228.5 \text{ mm}$$



- Calculate the distance $\overline{AF}$. (2 P.)
- Calculate the perimeter of the triangle AHE . (2 P.)
- Calculate the size of the angle δ . (1 P.)